3 The case for $k = 2$

By using congruence, we can prove Theorem 3.1.

Theorem 3.1 *The Diophantine equation $x^2 + y^2 + z^2 = 2xyz$ has no positive interger solution.*

Proof. It is easy to see that there are only two situations of the solutions. We can divide the solutions into two cases.
Case 1. One entry of the solution is even and the others are odd.
Thus $a^2 + b^2 + c^2 \equiv 2 \pmod 4$ and $2abc \equiv 0 \pmod 4$, a contradiction.
Case 2. The three entries of the solution are even.
Let (a, b, c) be a solution and $a = 2l,\ b = 2m,\ c = 2n,\ l,\ m,\ n \in \mathbf{N}$. We can get $l^2 + m^2 + n^2 = 4lmn$. By Theorem 2.1, a contradiction. $\square$