5 The case for $k = 3$

The Markoff Conjecture has become known widely when Cassels [4] mentioned it in 1957. Up to now, the Markoff Conjecture has been proved only for some special cases. Here we list some results.

Baragar [1] proved Theorem 5.1 by using algebraic number theory.

Theorem 5.1 *If either c, $3c-2$ or $3c+2$ is a prime, twice a prime or four times a prime, then there exists at most one integer pair (a, b) so that (a, b, c) is a Markoff triple.*

There are four ways to prove Theorem 5.2. Button [2] proved it by using algebraic number theory. Schmutz [8] proved it by using hyperbolic geometry. Lang and Tan [7] proved it by using some elementary facts from the hyperbolic geometry of the modular torus with one cusp. Zhang [9] presented a very elementary proof by using neither algebraic number theory nor hyperbolic geometry.

Theorem 5.2 *A Markoff triples up to permutation is determined uniquely by its largest entry if the latter is a prime power or twice a prime power.*

Zhang [10] proved the following theorem by using some simple facts about congruence.

Theorem 5.3 *A Markoff triples up to permutation is determined uniquely by c if c satisfies:*
(i) $3c+2$ or $3c-2$ is a prime power when c is odd, or
(ii) $3c-2$ is 4 times a prime power or $3c+2$ is 8 times a prime power when c is even.