On the Behavior of Electromagnetic Field  
At Twisted Edges in Inhomogeneous Medium

You-Jen Wu

Department of Physics  
College of Sciences

Abstract

The behavior of electromagnetic fields near a twisted edge in inhomogeneous medium is considered, starting from the condition that the energy density must be integrable over any finite domain. Differential equations of Sturm-Liouville type are then set up, which, together with relevant boundary conditions, determine the eigenvalues and accordingly the field behavior. Static fields are shown to have the same behavior in the neighborhood of such geometrical singularities. Media with piecewise constant permeability and permittivity are discussed as application examples of the concept of "impedance" or "admittance" for solving the associated eigenvalue problems.

I. INTRODUCTION

The edge condition was introduced by Meixner in his investigations of diffraction of electromagnetic waves by a planar circular disc to assure the uniqueness of solution, although some earlier investigators seemed to have already been aware of it. Some generalisations to the case of a number of magnetodielectric wedges with a common edge have been recently reported. The case of a twisted conductor edge in a homogeneous space has also been treated. In a sense, however, these are special cases of a more general class of classical Sturm-Liouville problems. Once the differential equations are suitably set up, they show the properties concerned more directly.
In this paper, the twisted edge in an inhomogeneous medium will be considered, differential equations of Sturm-Liouville type will set up, and the solution to associated eigenvalue problem will be described with simple example.

II. TWISTED EDGE

Assume at first that an inhomogeneous medium with arbitrary permeability \( \mu(r) \) and permittivity \( \varepsilon(r) \) and a conductor have a common edge \( k \), which in turn is a curve in the three-dimensional space. As parameter we choose the arc length \( S \) to describe this curve.

On doing this, the curvature \( \kappa(s) \), torsion \( \lambda(s) \), and the trihedron \( t(s), n(s), b(s) \), (i.e. tangent, nor, and binormal vector respectively) at each point on the curve are determined (Fig. 1). In addition to the arc length \( s \), we introduce in the normal plane another two coordinates \( \theta \) and \( \varphi \) as shown in Fig. 1 in order to completely describe the space points near the edge. For point lying near enough to the edge this description is unique in case of \( \kappa < \infty \). But \( \theta, \varphi, s \) so chosen do not generally construct a system of orthogonal curvilinear coordinates. To be such one we must choose \( \theta, \xi, S \) instead, where

\[
\xi = \xi(s) = \varphi + \int_{s_0}^s \lambda(s) \, ds
\]

(see Appendix). In this coordinate system the divergence of a vector field \( A \) can be written as

\[
\nabla \cdot A = \frac{1}{\varepsilon(1 - \kappa \varepsilon \cos \varphi)} \left\{ \frac{\partial}{\partial \theta} \left( 1 - \kappa \varepsilon \cos \varphi \right) A_\theta + \frac{\partial}{\partial \xi} \left( 1 - \kappa \varepsilon \cos \varphi \right) A_\xi + \frac{\partial}{\partial S} \varepsilon A_S \right\}
\]

while the curl of a vector field \( A \) has the components

\[
\nabla \times A |_e = \frac{1}{\varepsilon(1 - \kappa \varepsilon \cos \varphi)} \left\{ \frac{\partial}{\partial \xi} \left( 1 - \kappa \varepsilon \cos \varphi \right) A_\theta - \frac{\partial}{\partial \theta} \varepsilon A_\xi \right\}
\]

\[
\nabla \times A |_\xi = \frac{1}{1 - \kappa \varepsilon \cos \varphi} \left\{ \frac{\partial}{\partial S} \varepsilon A_\theta - \frac{\partial}{\partial \varepsilon} (1 - \kappa \varepsilon \cos \varphi) A_\xi \right\}
\]

\[
\nabla \times A |_S = \frac{1}{\varepsilon} \left\{ \frac{\partial}{\partial \varepsilon} e A_\xi - \frac{\partial}{\partial \xi} A_\varepsilon \right\}
\]
The volume element is
\[ d\mathbf{V} = d\xi \cdot d\eta \cdot d\zeta \cdot \sqrt{1 - \varepsilon \zeta \cos \varphi} \, ds = \varepsilon (1 - \varepsilon \zeta \cos \varphi) \, d\xi \, d\eta \, d\zeta \, ds. \]

Equations (2) to (6) are valid for \( \varepsilon \cos \varphi < 1/\zeta \). Taking in mind that we are interested in the behavior of electromagnetic fields only in the neighborhood of the edge (\( \varepsilon \to 0 \)) and the fact that the volume element \( d\mathbf{V} \) includes a factor \( \varepsilon \), we may expand the field components according to the edge condition as follows:

\[
\begin{align*}
E_\varphi (\varepsilon, \xi, s) &= \varepsilon^{2-1} [a_0(\xi, s) + a_1(\xi, s)\varepsilon + \ldots] = \varepsilon E_\varphi (\varepsilon, \xi, s) + \ldots \\
E_\eta (\varepsilon, \xi, s) &= \varepsilon^{2-1} [b_0(\xi, s) + b_1(\xi, s)\varepsilon + \ldots] = \varepsilon E_\eta (\varepsilon, \xi, s) + \ldots \\
E_\zeta (\varepsilon, \xi, s) &= \varepsilon^{2-1} [c_0(\xi, s) + c_1(\xi, s)\varepsilon + \ldots] = \varepsilon E_\zeta (\varepsilon, \xi, s) + \ldots \\
H_\varphi (\varepsilon, \xi, s) &= \varepsilon^{2-1} [d_0(\xi, s) + d_1(\xi, s)\varepsilon + \ldots] = \varepsilon H_\varphi (\varepsilon, \xi, s) + \ldots \\
H_\eta (\varepsilon, \xi, s) &= \varepsilon^{2-1} [e_0(\xi, s) + e_1(\xi, s)\varepsilon + \ldots] = \varepsilon H_\eta (\varepsilon, \xi, s) + \ldots \\
H_\zeta (\varepsilon, \xi, s) &= \varepsilon^{2-1} [f_0(\xi, s) + f_1(\xi, s)\varepsilon + \ldots] = \varepsilon H_\zeta (\varepsilon, \xi, s) + \ldots
\end{align*}
\]

where \( \zeta > 0 \) and \( e_\varphi (\varepsilon, \xi, s) = \varepsilon^{2-1} a_0(\xi, s) \) is the first term in the expansion of \( E_\varphi (\varepsilon, \xi, s) \) with analogues for \( E_\eta (\varepsilon, \xi, s), \ldots, H_\zeta (\varepsilon, \xi, s) \).

Inserting (7) and (8) into Maxwell's divergence equations \( \nabla \cdot D = 0 \) and \( \nabla \cdot B = 0 \) and reserving only the principal terms, we have

\[
\varepsilon \frac{\partial}{\partial \varepsilon} \varepsilon E + \frac{\partial}{\partial \xi} E \xi = 0 \quad \text{(9)} \quad \mu \frac{\partial}{\partial \varepsilon} \mu H + \frac{\partial}{\partial \xi} \mu H \xi = 0 \quad \text{(10)}
\]

where we have assumed that
\[
\frac{\partial}{\partial \varepsilon} \varepsilon = 0 \quad \text{and} \quad \frac{\partial}{\partial \varepsilon} \mu = 0
\]
Likewise, the insertion of (7) and (8) into the s-component of Maxwell's curl equations

\[
j \omega \varepsilon E = \nabla \times H \quad \text{and} \quad -j \omega \mu H = \nabla \times E
\]
gives the principal term relations:

\[
\frac{\partial}{\partial \varepsilon} \varepsilon H \xi - \frac{\partial}{\partial \xi} H \xi = 0 \quad \text{(11)}
\]
\[
\frac{\partial}{\partial \varepsilon} \varepsilon E \xi - \frac{\partial}{\partial \xi} E \xi = 0 \quad \text{(12)}
\]
Other relations may be obtained from the \( \varepsilon \) - and \( \xi \) - component of the curl equations:

\[
\frac{\partial}{\partial \xi} h_s = 0 \quad \text{(13)}
\]
The set of eight scalar Eq. (9) to (16) is to be satisfied simultaneously. This can be divided into two simpler cases:

(A) \( h_e = h_\xi = 0 \) while \( e_e \) and \( e_\xi \) are not identically zero. We denote this as “dominant electric” (D. E.) type.

(B) \( e_e = e_\xi = 0 \) while \( h_e \) and \( h_\xi \) are not identically zero. We denote this as “dominant magnetic” (D. M.) type.

For the first case, we have, after eliminating from (9) and (12), the differential equation for the field component \( e_e \):

\[
\frac{1}{\xi} \frac{\partial}{\partial \xi} \left( \xi \frac{\partial e_e}{\partial \xi} \right) + \tau^2 e_e = 0 \tag{17}
\]

The other field components are given by (12) and (13) to (16). For the second case we have similarly the differential equation:

\[
\frac{1}{\mu} \frac{\partial}{\partial \xi} \left( \mu \frac{\partial h_e}{\partial \xi} \right) + \tau^2 h_e = 0 \tag{18}
\]

with the other field components given by (11) and (13) to (16).

It is clear that, Eqs. (17) and (18) are of Sturm-Liouville type, of which many properties are well known. One important property is that both the solution, say, \( e_e \) and the function \( \xi (\partial e_e / \partial \xi) \) are continuous even when the coefficient function \( \xi (\xi, s) \) has discontinuities. If we define \( e_e \) (or \( \mu(\partial h_e / \partial \xi) \)) as “voltage” and \( \xi (\partial e_e / \partial \xi) \) (or \( h_e \)) as “current”, then the “impedance”

\[ e_e / \xi (\partial e_e / \partial \xi) \quad \text{or} \quad \mu (\partial h_e / \partial \xi) / h_e \]

or, perhaps for another convenience, the “admittance”

\[ \xi (\partial e_e / \partial \xi) / e_e \quad \text{or} \quad h_e / (\mu (\partial h_e / \partial \xi)) \]

of the solution is continuous. This concept will be found of utility when \( \xi (\xi, s) \) and \( \mu (\xi, s) \) are piecewise constant function of \( \xi \). Eqs. (17) and (18), when subject to boundary conditions, form two eigenvalue problem for the eigenvalue \( \tau(s) \). The set \( \xi \) of eigenvalues from (17) need not be the same as the set \( \mu \) from (18). Further considerations give the following general result about the behavior of field for \( e \to 0 \) which we put in tabular form (see Table 1 and Table 2).
On the Behavior of Electromagnetic Field At Twisted Edges in Inhomogeneous Medium

Generally, the fields are a combination of those of D. E. type and those of D. M. type. As far as the singular behavior of the fields is concerned, we must choose for each field component the more singular one from both types. From Table 2 it is clear that transversal components of electromagnetic fields may be singular whereas the longitudinal components $E_s$, $H_s$ are regular and may even have finite values. There are also circumstances in which all the field components are finite near the edge ($\tau_{\mathcal{E}} > 1$, $\tau_{\mathcal{M}} > 1$).

The divergence Equations $\nabla \cdot D = 0$ and $\nabla \cdot B = 0$ are valid in the electrostatic or magnetostatic case; the two Equations (9) and (10) come from the general expression (2) for the divergence of a vector field by comparing the coefficients of equal powers of $e$. Furthermore, the six Eqs. (11) to (16) are also the result of the same procedure, which gives the very same equations no matter whether $\omega = 0$ or not. This means that all our results are valid for both electromagnetic and static cases.

Table 1

<table>
<thead>
<tr>
<th>$e_{\mathcal{E}}$</th>
<th>$e_{\mathcal{M}}$</th>
<th>$e_s$</th>
<th>$h_{\mathcal{E}}$</th>
<th>$h_{\mathcal{M}}$</th>
<th>$h_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>D. E.</td>
<td>Eq. (17)</td>
<td>Eq. (12)</td>
<td>O (if $1 \notin S_{\mathcal{E}}$)</td>
<td>O (if $1 \notin S_{\mathcal{M}}$)</td>
<td>O (if $1 \notin S_{\mathcal{E}}$)</td>
</tr>
<tr>
<td>b. c.</td>
<td>O</td>
<td>C_{o(s)} O (if $1 \notin S_{\mathcal{E}}$)</td>
<td>O (if $1 \notin S_{\mathcal{M}}$)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2

<table>
<thead>
<tr>
<th>$E_{\mathcal{E}}$</th>
<th>$E_{\mathcal{M}}$</th>
<th>$E_s$</th>
<th>$H_{\mathcal{E}}$</th>
<th>$H_{\mathcal{M}}$</th>
<th>$H_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>D. E.</td>
<td>$O (e^{\mathcal{E}})$</td>
<td>$O (e^{\mathcal{E}})$</td>
<td>$O (e^{\mathcal{E}})$</td>
<td>$O (e^{\mathcal{E}})$</td>
<td>$O (e^{\mathcal{E}})$</td>
</tr>
<tr>
<td>D. M.</td>
<td>$O (e^{\mathcal{M}})$</td>
<td>$O (e^{\mathcal{M}})$</td>
<td>$O (e^{\mathcal{M}})$</td>
<td>$O (e^{\mathcal{M}})$</td>
<td>$O (e^{\mathcal{M}})$</td>
</tr>
</tbody>
</table>

There is one more thing to be mentioned. Because of the torsion $\lambda(s)$ of the edge, the angle $\xi$ must be measured from some plane which varies from point to point along the edge and is in general different from the local osculating plane. However, as shown by (17) and (18), the eigenvalues $\tau (s)$, and therefore the local singular behavior of the fields near the edge, are determined only by the local configuration of the medium in the normal plane. In the plane passing through some fixed point $P$ and normal to the edge at point $Q$, $s$=some fixed value, $d \xi = d \varphi$, the eigenvalues $\tau (s)$ are independent of angle-measuring, as...
it must be. Thus serves only as a parameter giving an account of the possible variation of local configurations of the medium in the normal planes along the twisted edge.

So far the local configuration \( \mu(\xi, s) \) and \( \varepsilon(\xi, s) \) is still arbitrary. As application we will now consider some examples.

### III EXAMPLES

Let \( N \) magnetodielectric wedges \( \mu_i(s), \varepsilon_i(s) \), each with wedge angle \( \psi_i(s) \), \( i = 1, 2, \ldots, N \) and a conductor wedge with wedge angle \( \psi_0(s) \) have a common edge (Fig. 2). The general solution \( e_i \) of Eq. (17) in the \( i \)-th medium is

\[
e_i e_i = e^{-1} \left( A_i \sin \zeta \xi + B_i \cos \zeta \xi \right)
\]

which, together with

\[
\varepsilon_i \frac{\partial e_i}{\partial \xi} = e^{-1} \varepsilon_i \left( A_i \cos \zeta \xi - B_i \sin \zeta \xi \right)
\]

is continuous over the boundaries separating different media. Note that \( A_i, B_i, \zeta, \xi, \) and \( z_i \) in (19) and (20) are functions of the parameters. The continuity condition at \( \xi = \xi_{i-1} \) now reads

\[
A_i \sin \zeta \xi_{i-1} + B_i \cos \zeta \xi_{i-1} = A_i \sin \zeta \xi_{i-1} + B_i \cos \zeta \xi_{i-1} \cos \xi_{i-1}
\]

\[
\varepsilon_i \left( A_i \cos \zeta \xi_{i-1} - B_i \sin \zeta \xi_{i-1} \right) = \varepsilon_{i-1} \left( A_i \cos \zeta \xi_{i-1} - B_i \sin \zeta \xi_{i-1} \right)
\]

By defining the “voltage” \( V \) the “current” \( I \), and the “input impedance” \( W \) of the solution \( e_i \) at the \( i \)-th “node” as

\[
V_i = A_i \sin \zeta \psi_i + B_i \cos \zeta \psi_i, \quad I_i = \varepsilon_i \left( A_i \cos \zeta \psi_i - B_i \sin \zeta \psi_i \right)
\]

\[W_i = \frac{V_i}{I_i}
\]

Equation (21) can after some manipulations be rewritten in the form of

\[
\begin{bmatrix} V_i \\ I_i \end{bmatrix} = \begin{bmatrix} \cos \zeta \psi_i & \frac{1}{\varepsilon_i} \sin \zeta \psi_i \\ -\varepsilon_i \sin \zeta \psi_i & \cos \zeta \psi_i \end{bmatrix} \begin{bmatrix} V_{i-1} \\ I_{i-1} \end{bmatrix} (i = 1, 2, \ldots, N)
\]

which in turn can be written as

\[
W_i = \frac{1}{\varepsilon_i} \frac{W_{i-1} + j \frac{1}{\varepsilon_i} \tan \psi_i}{\frac{1}{\varepsilon_i} + j W_{i-1} \tan \psi_i}
\]
On the Behavior of Electromagnetic Field At Twisted Edges in Inhomogeneous Medium

a familiar formula for the impedance transformation in transmission line theory, such that the Smith chart can be used as well in the case of twisted edges. \( \psi_1, \psi_2 \) and \( W_1 \) here are functions of the arc length parameter \( s \).

Formula (22b) together with boundary conditions, namely \( W_0 = 0 \) at \( \xi = \xi_0 = \xi_a \), and \( W_N = 0 \) at \( \xi = \xi_N = \xi_e \), can be used to determine the set \( S_\xi \) of eigenvalues \( \tau \). To be specific, we have

\[
W = 0
\]

\[
W_1 = j \frac{1}{E_1} T_1
\]

\[
W_2 = j \frac{E_1 T_1 + E_2 T_2}{1 - \frac{E_2}{E_1} T_1 T_2}
\]

\[
W_3 = \frac{j}{1 - \frac{E_1 T_1 + E_2 T_2 + E_3 T_3}{E_1 T_1 T_2 + E_2 T_2 T_3 + E_3 T_3}} \frac{E_1 T_1 T_2 T_3}{1 - \frac{E_1 T_1 + E_2 T_2 + E_3 T_3}{E_1 T_1 T_2 + E_2 T_2 T_3 + E_3 T_3}}
\]

\[
W_4 = j \frac{E_1 T_1 + E_2 T_2 + E_3 T_3 + E_4 T_4}{1 - \frac{E_1 T_1 + E_2 T_2 + E_3 T_3 + E_4 T_4}{E_1 T_1 T_2 + E_2 T_2 T_3 + E_3 T_3 T_4 + E_4 T_4}} \frac{E_1 T_1 T_2 T_3 T_4}{1 - \frac{E_1 T_1 + E_2 T_2 + E_3 T_3 + E_4 T_4}{E_1 T_1 T_2 + E_2 T_2 T_3 + E_3 T_3 T_4 + E_4 T_4}}
\]

\[
W_{2m} = \sum \frac{1}{E_1} \frac{E_2}{E_1 E_3} T_1 T_2 T_3 + \cdots + (-1)^{m-1} \frac{E_2}{E_1 E_3 \cdots E_{2m-1}} T_1 T_2 T_3 + \cdots + (-1)^{m-1} \frac{E_2}{E_1 E_3 \cdots E_{2m-1}} T_1 T_2 T_3
\]

\[
W_{2m+1} = \sum \frac{1}{E_1} \frac{E_2}{E_1 E_3} T_1 T_2 T_3 + \cdots + (-1)^m \frac{E_2}{E_1 E_3 \cdots E_{2m+1}} T_1 T_2 T_3
\]

\[
W_N = 0
\]

where \( \psi_1 = \tan \tau \psi_1 \) and \( \Sigma \) means the combination of all the same forms without permutation. From these we see that for \( N = 2m + 1 \) the set \( S_\xi \) of eigenvalues is obtained from

\[
\tan \psi_1 = 0 \quad (N = 1)
\]

\[
\frac{1}{E_1} \tan \psi_1 + \frac{1}{E_2} \tan \psi_2 + \frac{1}{E_3} \tan \psi_3 - \frac{E_2}{E_1 E_3} \tan \psi_1 \tan \psi_2 \tan \psi_3 = 0 \quad (N = 3)
\]

\[
\Sigma \frac{1}{E_1} T_1 - \sum \frac{E_2}{E_1 E_3} T_1 T_2 T_3 + \cdots + (-1)^m \frac{E_2 E_3 \cdots E_{2m+1}}{E_1 E_3 \cdots E_{2m+1}} T_1 T_2 T_3 = 0
\]

\[
(N = 2m + 1) \quad (26)
\]

For \( N = 2m \) the eigenvalues can be calculated by equating the numerator to zero as before resulting in the set \( S_\xi \). But in case of identical wedge angles \( \psi_1 = \psi_2 = \psi_3 \), 

\[
-517-
\]
there is another set $S''$ of eigenvalues by setting $\tan \tau \psi_1 = \infty$; that is,

$$s'' = \left\{ \tau \mid \tau \psi_1 = \frac{2n+1}{2}, \ N = 0, 1, 2 \ldots \right\}$$

and the total set of eigenvalues is $S = S'_e \cup S''$. For the set $S'_e$ we have explicitly

$$\frac{1}{E_1} \tan \tau \psi_1 + \frac{1}{E_2} \tan \tau \psi_2 = 0 \quad \text{(N = 2)}$$

$$\frac{1}{E_1} T_1 + \frac{1}{E_2} T_2 + \frac{1}{E_3} T_3 + \frac{1}{E_4} T_4 - \left( \frac{E_2}{E_1 E_3} T_1 T_2 T_3 + \frac{E_2}{E_1 E_4} T_1 T_2 T_4 + \frac{E_3}{E_1 E_4} T_3 T_4 \right) = 0 \quad \text{(N = 4)}$$

$$\sum \frac{1}{E_1} T_i = \sum \frac{E_2}{E_1 E_3} T_1 T_2 T_3 + \ldots + (-1)^{m-1} \sum \frac{E_2 E_4}{E_1 E_3 \ldots E_{2m-4}} T_1 T_2 \ldots T_{2m-1} \equiv 0 \quad \text{(N = 2m)} \quad (27)$$

So far we have discussed only the differential Equation (17). The treatment of Eq. (18) is essentially the same except following minor modifications:

$$I_i = A_i \sin \tau \xi_i + B_i \cos \tau \xi_i$$

$$\overline{V}_i = \mu_i A_i \cos \tau \xi_i \quad B_i \sin \tau \xi_i$$

$$Y_i = j \frac{1}{\bar{V}_i} = \frac{1}{W} \quad (28)$$

$$W_i = \mu_i \frac{W_{i-1} + j \mu_i \tan \tau \psi_i}{W_{i-1} + j \mu_i \tan \tau \psi_i} \quad (29)$$

Thus all things are valid if one writes $\mu_i$ instead of $1/E_i$ in Eqs. (22b) to (27).

Some special cases were given in [7] and will not be repeated here. If the conductor wedge does not appear, the solution procedure (22a) still holds if we now require $V_N = V_0$ and $I_N = I_0$. This leads to transcendental equations for $\tau$. A simple example for $N = 2$ was given in [6].

**APPENDIX**

We will show by construction that the set $\vartheta, \xi, s$ can be used as an orthogonal curvilinear coordinate system. For every point $P$ lying near enough to the edge, more precisely, for every space point $P$ with $\vartheta < 1/\pi, \varphi < \infty$, we can uniquely choose a plane normal to the edge and passing through the point (Fig. 3). This normal plane is spanned by the normal vector $n(s)$ and the binormal vector $b(s)$ for some $s$. Thus $s$ serves as a coordinate to describe the point $P$.

Let other two coordinates $\vartheta$ and $\varphi$ be introduced in the normal plane as shown.
On the Behavior of Electromagnetic Field At Twisted Edges in Inhomogeneous Medium

in the Figure. It is easy to check that $e$, $\varphi$, $s$ so chosen do not generally form a systes of orthogonal curvilinear coordinates. Instead of $\varphi$, we introduce therefore another angle $\xi := \varphi + \theta(s)$ with corresponding unit vector $\nu(s)$ and $\omega(s)$. The position vector for the point $P$ can be expressed as

$$\gamma(\alpha \beta s) = x(s) + \alpha \nu(s) + \beta \omega(s)$$

where $x(s)$ is the position vector for the point $Q$ on the edge and $\alpha := e \cos \xi$, $\beta := e \sin \xi$

The orthogonality of the three vectors requires that

\[ i \cdot \omega(s) = 0 \tag{A1} \]

where subscript $\alpha$ denotes partial derivative with respect to $\alpha$ and the prime indicates derivative with respect to $s$. Since

$$\nu(s) = \cos \theta(s) n(s) - \sin \theta(s) b(s) \tag{A2}$$

$$\omega(s) = \sin \theta(s) n(s) + \cos \theta(s) b(s)$$

it follows from (A1) and (A2) that

$$\theta'(s) - \lambda(s) = 0 \tag{A3}$$

where Serret-Frenet formulas have been made use of.

Thus we have

$$\xi = \varphi + \theta'(s) = \varphi + \int_s^\xi \lambda(s)$$

The expressions (2) to (6) in Section 2 are then a direct consequence of the orthogonality of this system.

References

1. Lord Rayleigh: *On the passage of waves through apertures in plane Screens, and allied problems*. Philos. Mag. 43 (1897) 259-272


不均勻介質中扭轉稜邊左近空間之電磁場

中文摘要

雖然，在早期文獻中對棱邊效應已有若干討論，直至 Meixner 研究電磁波衍射現象時，方始對邊稜條件加以界定。晚近，有關數種不同介質具有共同棱稜之情況續有專文報導；而在均勻介質中扭轉邊稜亦有研究成果。適當的微分方程一經列出，所探察之結果隨之顯現。

本文係探討在不均勻介質中扭轉稜邊左近電磁場之形態。但探討條件限於能量密度在一定區域內爲具有可積分性者。列出 Sturm-Liouville 式微分方程，而以簡單例題表出具有本徵值之解。