5 The case for $k = 3$

The Markoff Conjecture has become known widely when Cassels [4] mentioned it in 1957. Up to now, the Markoff Conjecture has been proved only for some special cases. Here we list some results.

Baragar [1] proved Theorem 5.1 by using algebraic number theory.

**Theorem 5.1** If either $c$, $3c - 2$ or $3c + 2$ is a prime, twice a prime or four times a prime, then there exists at most one integer pair $(a, b)$ so that $(a, b, c)$ is a Markoff triple.


**Theorem 5.2** A Markoff triples up to permutation is determined uniquely by its largest entry if the latter is a prime power or twice a prime power.

Zhang [10] proved the following theorem by using some simple facts about congruence.

**Theorem 5.3** A Markoff triples up to permutation is determined uniquely by $c$ if $c$ satisfies:

(i) $3c + 2$ or $3c - 2$ is a prime power when $c$ is odd, or
(ii) $3c - 2$ is 4 times a prime power or $3c + 2$ is 8 times a prime power when $c$ is even.